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Control in the operational space of bilateral teleoperators with time-delays and without velocity measurements ⋆

Carlos I. Aldana * Emmanuel Cruz * Emmanuel Nuño *
Luis Basañez **

* *Department of Computer Science, CUCEI, University of Guadalajara, 44430 Guadalajara, JAL, Mexico. (e-mails: ivan.aldana@academicos.udg.mx, emitacz@yahoo.com.mx, emmanuel.nuno@cucei.udg.mx).*
** *Institute of Industrial and Control Engineering, Technical University of Catalonia. Barcelona 08028, Spain. luis.basanez@upc.edu.*

Abstract: This paper proposes a control scheme in the operational space for bilateral teleoperation systems composed of heterogeneous robots (kinematically and dynamically different) without velocity sensors and considering variable time-delays in the interconnection. The proposed control scheme use a second order dynamical controller that back-propagates damping to the local and the remote manipulators. Under the assumptions that the human operator and the environment define passive maps from force to velocity, it is proved that velocities and pose (position and orientation) errors between the local and the remote manipulators are bounded. Moreover, in the case that the human and the environment forces are zero, the velocities and pose errors converge asymptotically to zero. The proposed approach employs, the singularity-free, unit-quaternions to represent the orientation of the end-effectors. The performance of the proposed controller is illustrated via simulations with a teleoperation system composed of robots with 3-DoF and 7-DoF.

Keywords: Operational Space, Teleoperation, Time-Delays, Euler-Lagrange systems, Nonlinear Control.

1. INTRODUCTION

The *operational space* control plays a major role in cooperative tasks performed by multi-robot systems, primarily if the robots are kinematically and dynamically dissimilar (heterogeneous). The operational space (also known as *task space*) is a subspace of the Special Euclidean space of dimension three, denoted $SE(3)$. It is well-known that the minimum number of coordinates required to define the pose (position and orientation) of an object in a three-dimensional space are six: three for the position and three for the orientation (Kelly et al., 2005). The drawback of the minimal orientation representations (*e.g.*, the Euler angles) is that they exhibit singularities. The *unit-quaternions* are an alternative non-minimal representation of orientation that have the key property to render a singularity-free representation which is the main motivation behind its use (Caccavale et al., 1999).

A *teleoperation system* (Fig. 1) can be seen as a particular case of a multi-robot system, since it is a type of network composed of two interconnected robots that interact with human and environmental forces. Teleoperation systems have a large area of application in special for the work in hazardous workspaces such as nuclear plants (Okura et al., 2013), outer space missions (Ge et al., 2014), etc. The communication channel generates a variety of constraints, like time-delays, that degrade the teleoperation system performance, and also may cause instability. An insightful historical survey that analyzes communication delays and other challenging conditions in teleoperation systems can be consulted in (Hokayem and Spong, 2006) and a tutorial of passivity-based controllers which guarantee stability for teleoperation systems with constant and variable time-delays is given in (Nuño et al., 2011).

Most of the teleoperation control schemes proposed in the literature require the knowledge of *velocity measurements* in their control laws. However, in general, velocity sensors are noisy and costly, and for this reason most of the robots do not have these sensors. Few remarkable exceptions are: In (Hua et al., 2015) is proposed a terminal-sliding-mode scheme to obtain an estimation of the manipulators velocities in finite-time and this estimation is used in a bounded proportional-derivative like controller to establish, after the solution of a LMI, asymptotic convergence of position error and velocities to zero. In (Sarras et al., 2016) the Immersion and Invariance (I&I) velocity observer has been ported to the bilateral teleoperators control. The main practical drawback of the las two works is the fact that their observer design requires exact knowledge of the complete system dynamics. In (Nuño et al., 2018) is proposed a dynamic controller robust to time-delays that does not rely on velocity measurements but it works only for kinematically similar robot manipulators.

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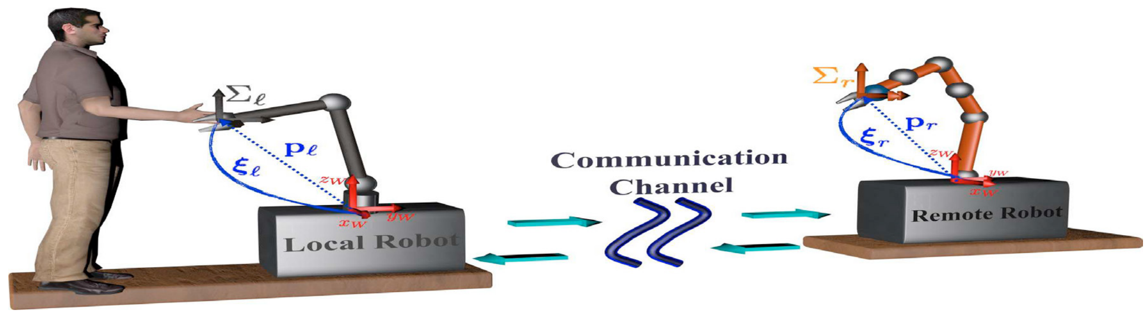


Fig. 1. Elements and coordinate frames of the teleoperation system.

All the previously mentioned works develop the control algorithms in the joint space. There are few works in the literature that propose controllers for teleoperation systems in the operational space. In (Wang and Xie, 2012) is proposed a control scheme robust to time-delays that achieve pose synchronization between the local and the remote robot manipulators but requires velocity measurementes; in (Ge et al., 2014) an adaptive regulator is added in the typical P+d controller to compensate for the unknown gravity signal, obtaining a control algorithm robust to time-delays in the communication but using a minimal orientation representation; in (Aldana et al., 2013) a velocity filter is proposed that solves the pose synchronization problem but without considering time-delays in the interconnection; in (Liu et al., 2014) is proposed a nonlinear adaptive controller for a teleoperation system with kinematic and dynamic uncertainties but do not consider time-delays in the communications and in the same way as most of the papers that work in the operational space, they use a minimal representation for the orientation.

This paper presents an extension to the operational space of the controller reported in (Nuño et al., 2018) that has been previously presented in the joint space for kinematically similar manipulators. Compared to the previous work, the present scheme can be used with heterogenous local and remote robots, moreover, the singularity-free unit-quaternions are employed to represent the orientation of the end–effectors. The proposed control scheme use a second order dynamical controller that back-propagates damping to the local and the remote manipulators. If sufficient damping is injected in the controller and under the common passivity assumption of the human operator and the environment, it is proved that pose errors and velocities are bounded. When the human and the environment do not inject forces in the system, it is shown that pose errors and velocities asymptotically converge to zero.

The following *notation* is used throughout the paper. $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{>0} := (0, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$. $|\mathbf{x}|$ stands for the standard Euclidean norm of vector $\mathbf{x}$. $\mathbf{I}_k$ represents the identity matrix of size $k \times k$. $\mathbf{1}_k$ and $\mathbf{0}_k$ represent column vectors of size k with all entries equal to one and to zero, respectively. For any function $\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the $\mathcal{L}_\infty$-norm is defined as $\|\mathbf{f}\|_\infty := \sup_{t\geq 0} |\mathbf{f}(t)|$, $\mathcal{L}_2$-norm as $\|\mathbf{f}\|_2 := (\int_0^\infty |\mathbf{f}(t)|^2 dt)^{1/2}$. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ spaces are defined as the sets $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_\infty < \infty\}$ and $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_2 < \infty\}$, respectively. The argument of all time dependent signals is omitted, *e.g.*, $\mathbf{x} \equiv \mathbf{x}(t)$, except for those which are time-delayed, *e.g.*, $\mathbf{x}(t - T(t))$.

2. TELEOPERATOR DYNAMICS AND KINEMATICS

The local and remote robot manipulators are modeled as a pair of n_i-DoF fully actuated, revolute joints, robots. Their Euler-Lagrange (EL) equations of motion, in joint space, are given by

$$\begin{aligned} \mathbf{M}_\ell(\mathbf{q}_\ell)\ddot{\mathbf{q}}_\ell + \mathbf{C}_\ell(\mathbf{q}_\ell, \dot{\mathbf{q}}_\ell)\dot{\mathbf{q}}_\ell + \mathbf{g}_\ell(\mathbf{q}_\ell) &= \boldsymbol{\tau}_h - \boldsymbol{\tau}_\ell \\ \mathbf{M}_r(\mathbf{q}_r)\ddot{\mathbf{q}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r + \mathbf{g}_r(\mathbf{q}_r) &= \boldsymbol{\tau}_r - \boldsymbol{\tau}_e, \end{aligned} \quad (1)$$

where $\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i \in \mathbb{R}^{n_i}$, $i \in \{\ell, r\}$, are the joint positions, velocities and accelerations, respectively; $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n_i \times n_i}$ is the symmetric and positive definite inertia matrix; $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{n_i \times n_i}$ is the Coriolis and centrifugal effects matrix; $\mathbf{g}_i(\mathbf{q}_i) \in \mathbb{R}^{n_i}$ is the gravitational torques vector; $\boldsymbol{\tau}_i \in \mathbb{R}^{n_i}$ is the control signal; $\boldsymbol{\tau}_h, \boldsymbol{\tau}_e \in \mathbb{R}^{n_i}$ are the joint torques induced by the human and environment forces respectively. Note that, because the robots can be heterogeneous, they might have different number of DoF; that is: $n_i \neq n_j$. The EL-system (1) possesses the following, well-known properties (Kelly et al., 2005):

P1. For all $\mathbf{q}_i \in \mathbb{R}^{n_i}$, there exist $\underline{M}_i, \bar{M}_i \in \mathbb{R}_{>0}$ such that $\underline{M}_i \leq \|\mathbf{M}_i(\mathbf{q}_i)\| \leq \bar{M}_i$. Further, $\mathbf{M}_i(\mathbf{q}_i) = \mathbf{M}_i^\top(\mathbf{q}_i)$.
P2. For all $\dot{\mathbf{q}}_i \in \mathbb{R}^{n_i}$, there exists $k_c \in \mathbb{R}_{>0}$ such that $|\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i| \leq k_c |\dot{\mathbf{q}}_i|^2$.
P3. Matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is skew-symmetric.

With regards to the human and the environment interactions, this paper makes the following standard assumption:

A1. There exists $\kappa_i \geq 0$ such that, $\forall t \geq 0$, $\mathcal{E}_i \leq \kappa_i < \infty$, the human operator and the environment define passive, velocity to force, maps, that is, $\forall t \geq 0$,

$$\mathcal{E}_l := \int_0^t \dot{\mathbf{q}}_l^\top(\sigma)\boldsymbol{\tau}_h(\sigma)d\sigma \leq \kappa_l, \ \mathcal{E}_r := -\int_0^t \dot{\mathbf{q}}_r^\top(\sigma)\boldsymbol{\tau}_e(\sigma)d\sigma < \kappa_r$$

With regards to the local-remote interconnection time-delays it is assumed that

A2. The information exchange is subject to a variable time-delay $T_i(t)$ with a known upper-bound ${}^*T_i$. Hence, it holds that $0 \leq T_i(t) \leq {}^*T_i < \infty$. Additionally, the time–derivative $\dot{T}_i(t)$ is bounded.

The pose of the i-th-end-effector, relative to a common reference frame, is denoted by the vector $\mathbf{x}_i \in \mathbb{R}^7$ and it contains the position vector $\mathbf{p}_i \in \mathbb{R}^3$ and the orientation given by the unit-quaternion[1] $\boldsymbol{\xi}_i \in S^3$, such that

[1] The set $S^3 \subset \mathbb{R}^4$ represents an unitary sphere of dimension three and it is defined as $S^3 := \{\boldsymbol{\xi} \in \mathbb{R}^4 : |\boldsymbol{\xi}|^2 = 1\}$.

$\mathbf{x}_i := [\mathbf{p}_i^\top, \boldsymbol{\xi}_i^\top]^\top$. The unit-quaternion is an orientation representation defined with four elements and, compared to other representations, it offers significant advantages in terms of numerical robustness and numerical singularities (Caccavale et al., 1999). A unit-quaternion $\boldsymbol{\xi}_i \in S^3$ is composed of two elements: one scalar term $\eta_i \in \mathbb{R}$ and one vectorial term $\boldsymbol{\beta}_i \in \mathbb{R}^3$. Thus $\boldsymbol{\xi}_i := [\eta_i, \boldsymbol{\beta}_i^\top]^\top$ and, from the unit norm constraint, $\eta_i^2 + \boldsymbol{\beta}_i^\top \boldsymbol{\beta}_i = 1$ (refer to (Chou, 1992) for a detailed list of properties and operations involving unit-quaternions). The unit-quaternion $\boldsymbol{\xi}_i$ can be obtained from the direct kinematics of each robot manipulator, via the rotation matrix $\mathbf{R}_i \in SO(3) := \{\mathbf{R}_i \in \mathbb{R}^{3\times3} \mid \mathbf{R}_i^\top \mathbf{R}_i = \mathbf{I}_3, \det(\mathbf{R}_i) = 1\}$ (Spong et al., 2005).

The orientation error, relative to the world frame, between two different frames, $\boldsymbol{\Sigma}_i$ and $\boldsymbol{\Sigma}_j$, can be described by the rotation matrix $\tilde{\mathbf{R}}_{ij} := \mathbf{R}_i \mathbf{R}_j^\top \in SO(3)$. The unit-quaternion describing such orientation error is given by

$$\tilde{\boldsymbol{\xi}}_{ij} = \boldsymbol{\xi}_i \odot \boldsymbol{\xi}_j^* = \begin{bmatrix} \tilde{\eta}_{ij} \\ \tilde{\boldsymbol{\beta}}_{ij} \end{bmatrix} = \begin{bmatrix} \eta_i \eta_j + \boldsymbol{\beta}_i^\top \boldsymbol{\beta}_j \\ -\mathbf{U}^\top(\boldsymbol{\xi}_i)\boldsymbol{\xi}_j \end{bmatrix}, \tag{2}$$

where $\odot$ is the quaternion product, $\boldsymbol{\xi}_{(\cdot)}^* = [\eta_{(\cdot)}, -\boldsymbol{\beta}_{(\cdot)}^\top]^\top$ is the conjugate of $\boldsymbol{\xi}_{(\cdot)}$, and $\mathbf{U}(\boldsymbol{\xi}_i)$ is defined as

$$\mathbf{U}(\boldsymbol{\xi}_i) := \begin{bmatrix} -\boldsymbol{\beta}_i^\top \\ \eta_i \mathbf{I}_3 - \mathbf{S}(\boldsymbol{\beta}_i) \end{bmatrix},$$

where $\mathbf{S}(\cdot)$ is the skew-symmetric matrix operator. [2]

The normality condition and some straightforward calculations show that $\tilde{\boldsymbol{\beta}}_{ij} = \mathbf{0}$ if and only if $\boldsymbol{\xi}_i = \pm \boldsymbol{\xi}_j$ (Aldana, 2015). This, in turn, implies that $\mathbf{U}^\top(\boldsymbol{\xi}_i)\boldsymbol{\xi}_j = \mathbf{0}_3$. A key observation is that $\boldsymbol{\xi}_i = \boldsymbol{\xi}_j$ and $\boldsymbol{\xi}_i = -\boldsymbol{\xi}_j$ represent the same physical orientation. The following properties have been borrowed from (Fjellstad, 1994; Chou, 1992) and are used throughout the rest of the paper.

P4. For all $\boldsymbol{\xi}_i \in S^3$, $\mathbf{U}^\top(\boldsymbol{\xi}_i)\mathbf{U}(\boldsymbol{\xi}_i) = \mathbf{I}_3$. Hence, $\mathrm{rank}(\mathbf{U}(\boldsymbol{\xi}_i)) = 3$ and $\ker(\mathbf{U}^\top(\boldsymbol{\xi}_i)) = \mathrm{span}(\boldsymbol{\xi}_i)$.
P5. For all $\boldsymbol{\xi}_i \in S^3$ and $\dot{\boldsymbol{\xi}}_i \in \mathbb{R}^4$, $\dot{\mathbf{U}}(\boldsymbol{\xi}_i) = \mathbf{U}(\dot{\boldsymbol{\xi}}_i)$.
P6. Since, for all $\boldsymbol{\xi}_i \in S^3$, $|\boldsymbol{\xi}_i| = 1$ then $\mathbf{U}(\boldsymbol{\xi}_i)$ is a bounded operator.

The kinematic relation between joint velocities and linear $\dot{\mathbf{p}}_i$ and angular $\boldsymbol{\omega}_i$ velocities of the i-th-end-effectors is given by

$$\boldsymbol{\upsilon}_i = [\dot{\mathbf{p}}_i^\top, \boldsymbol{\omega}_i^\top]^\top = \mathbf{J}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i, \tag{3}$$

where $\boldsymbol{\upsilon}_i \in \mathbb{R}^6$ and $\mathbf{J}_i(\mathbf{q}_i) \in \mathbb{R}^{6\times n_i}$ is the *geometric Jacobian* matrix and it has the following property,

P7. For all $\mathbf{q}_i \in \mathbb{R}^{n_i}$, the Jacobian matrix $\mathbf{J}_i(\mathbf{q}_i)$ is a bounded operator.

The relation between the time-derivative of the unit-quaternion and the angular velocity, relative to the world reference frame, is given by

$$\dot{\boldsymbol{\xi}}_i = \frac{1}{2}\mathbf{U}(\boldsymbol{\xi}_i)\boldsymbol{\omega}_i. \tag{4}$$

Hence, defining $\boldsymbol{\Phi}(\boldsymbol{\xi}_i) := \mathrm{diag}(\mathbf{I}_3, \frac{1}{2}\mathbf{U}(\boldsymbol{\xi}_i))$, it holds that

$$\dot{\mathbf{x}}_i = \boldsymbol{\Phi}(\boldsymbol{\xi}_i)\boldsymbol{\upsilon}_i. \tag{5}$$

[2] For any $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3$, $\mathbf{S}(\mathbf{a})\mathbf{b} = \mathbf{a} \times \mathbf{b}$. Some well known properties of the skew-symmetric matrix operator, $\mathbf{S}(\cdot)$, used throughout the paper are: $\mathbf{S}(\mathbf{a})^\top = \mathbf{S}(-\mathbf{a}) = -\mathbf{S}(\mathbf{a})$ and $\mathbf{S}(\mathbf{a})\mathbf{a} = \mathbf{0}_3$.

3. PROPOSED CONTROLLER

3.1 Problem Statement

This paper considers a bilateral teleoperator system modeled by (1) that can be composed of heterogeneous robots. It is assumed that the interconnection time-delays satisfy assumption **A2** and that the velocity measurements of the robots are not available. The control objective is to design control laws $\boldsymbol{\tau}_i$, such that the following is fulfilled:

a) If the human and the environment satisfy **A1**, velocities and pose error are bounded, i.e.,
$$\boldsymbol{\upsilon}_i,\ |\mathbf{x}_l - \mathbf{x}_r| \in \mathcal{L}_\infty.$$
b) If the human and the environment forces are zero, i.e., $\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$, velocities and pose error asymptotically converge to zero, i.e.,
$$\lim_{t\to\infty} \boldsymbol{\upsilon}_i(t) = \mathbf{0}, \quad \lim_{t\to\infty} (\mathbf{x}_l(t) - \mathbf{x}_r(t)) = \mathbf{0}.$$

3.2 Proposed Solution

In order to solve the teleoperation problem previously described, the following dynamic controllers for the local and remote robots are proposed:

$$\begin{aligned} \boldsymbol{\tau}_l &= -\mathbf{g}_l(\mathbf{q}_l) + k_l \mathbf{J}_l^\top(\mathbf{q}_l)\boldsymbol{\Phi}^\top(\boldsymbol{\xi}_l)(\mathbf{x}_l - \boldsymbol{y}_l), \\ \boldsymbol{\tau}_r &= \mathbf{g}_r(\mathbf{q}_r) - k_r \mathbf{J}_r^\top(\mathbf{q}_r)\boldsymbol{\Phi}^\top(\boldsymbol{\xi}_r)(\mathbf{x}_r - \boldsymbol{y}_r), \end{aligned} \tag{6}$$

where $k_i \in \mathbb{R}_{>0}$ and $\boldsymbol{y}_i := [\mathbf{p}_{y,i}^\top, \boldsymbol{\xi}_{y,i}^\top]^\top \in \mathbb{R}^7$ is obtained as in (5), such that $\dot{\boldsymbol{y}}_i = \boldsymbol{\Phi}(\boldsymbol{\xi}_{y,i})\dot{\boldsymbol{\theta}}_i$, where $\dot{\boldsymbol{\theta}}_i \in \mathbb{R}^6$ is the vector of the linear and the angular velocities of the controller coordinates. The controller dynamics is defined as the following second order system, for $i, j \in \{\ell, r\}$ and $i \neq j$,

$$\ddot{\boldsymbol{\theta}}_i = -k_i \boldsymbol{\Phi}^\top(\boldsymbol{\xi}_{y,i})(\boldsymbol{y}_i - \mathbf{x}_i) - d_i \dot{\boldsymbol{\theta}}_i - \gamma_i \boldsymbol{\Phi}^\top(\boldsymbol{\xi}_{y,i})(\boldsymbol{y}_i - \boldsymbol{y}_j(t - T_j(t))), \tag{7}$$

where $d_i, \gamma_i \in \mathbb{R}_{>0}$ are the damping and proportional gains.

Note that the local–remote interconnection is located at the controller's dynamics and using the controllers states not at the robot's dynamics as it is commonly done (Nuño et al., 2008; Niemeyer and Slotine, 1991; Lee and Spong, 2006), this feature allows the robustness to time-delays without measuring velocities (Nuño et al., 2018). In this case, damping is injected in the controller dynamics and it back propagates to the robot, this methodology follows the idea of interconnection damping assignment in passivity–based control (Ortega et al., 1998).

The complete closed-loop system is

$$\nabla_l \begin{cases} \ddot{\mathbf{q}}_l = -\mathbf{M}_l^{-1}(\mathbf{q}_l)\mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\dot{\mathbf{q}}_l \\ \qquad - \mathbf{M}_l^{-1}(\mathbf{q}_l)\left(k_l \mathbf{J}_l^\top(\mathbf{q}_l)\boldsymbol{\Phi}^\top(\boldsymbol{\xi}_l)(\mathbf{x}_l - \boldsymbol{y}_l) - \boldsymbol{\tau}_h\right), \\ \ddot{\boldsymbol{\theta}}_l = -k_l \boldsymbol{\Phi}^\top(\boldsymbol{\xi}_{y,l})(\boldsymbol{y}_l - \mathbf{x}_l) - d_l \dot{\boldsymbol{\theta}}_l \\ \qquad - \gamma_l \boldsymbol{\Phi}^\top(\boldsymbol{\xi}_{y,l})(\boldsymbol{y}_l - \boldsymbol{y}_r(t - T_r(t))), \\ \dot{\boldsymbol{y}}_l = \boldsymbol{\Phi}(\boldsymbol{\xi}_{y,l})\dot{\boldsymbol{\theta}}_l, \end{cases}$$
$$\nabla_r \begin{cases} \ddot{\mathbf{q}}_r = -\mathbf{M}_r^{-1}(\mathbf{q}_r)\mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r \\ \qquad - \mathbf{M}_r^{-1}(\mathbf{q}_r)\left(k_r \mathbf{J}_r^\top(\mathbf{q}_r)\boldsymbol{\Phi}^\top(\boldsymbol{\xi}_r)(\mathbf{x}_r - \boldsymbol{y}_r) + \boldsymbol{\tau}_e\right), \\ \ddot{\boldsymbol{\theta}}_r = -k_r \boldsymbol{\Phi}^\top(\boldsymbol{\xi}_{y,r})(\boldsymbol{y}_r - \mathbf{x}_r) - d_r \dot{\boldsymbol{\theta}}_r \\ \qquad - \gamma_r \boldsymbol{\Phi}^\top(\boldsymbol{\xi}_{y,r})(\boldsymbol{y}_r - \boldsymbol{y}_l(t - T_l(t))), \\ \dot{\boldsymbol{y}}_r = \boldsymbol{\Phi}(\boldsymbol{\xi}_{y,r})\dot{\boldsymbol{\theta}}_r. \end{cases} \tag{8}$$

Now we are ready to state the main result of this paper.

Proposition 1. The local and the remote controllers defined in (6), with dynamics (7), solve the control objective a). Besides, these controllers also solve the control objective b) everywhere, except at $(\boldsymbol{\upsilon}_l(0), \mathbf{p}_l(0), \xi_l(0)) = (0_7, \mathbf{p}_r, -\xi_r)$. In both cases, the damping and the proportional gains satisfy the following condition

$$4d_l d_r > ({}^*T_l + {}^*T_r)^2 \gamma_l \gamma_r. \tag{9}$$

Proof. For Part a), We define $\mathcal{W}_i$ as the energy of the robot, the energy supplied by the human/environment plus the energy of the controller and the robot-controller interconnection, for $i, j \in \{\ell, r\}$ and $i \neq j$,

$$\mathcal{W}_i := \frac{1}{2}\dot{\mathbf{q}}_i^\top \mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i - \mathcal{E}_i + \kappa_i + \frac{1}{2}|\dot{\boldsymbol{\theta}}_i|^2 + k_i\frac{1}{2}|\mathbf{x}_i - \boldsymbol{y}_i|^2,$$

Assumption **A1** ensures that $\kappa_i - \mathcal{E}_i \geq 0$. Further, after differentiation it yields

$$\begin{aligned}\dot{\mathcal{W}}_i =& \dot{\mathbf{q}}_i^\top \mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \frac{1}{2}\dot{\mathbf{q}}_i^\top \dot{\mathbf{M}}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i - \dot{\mathcal{E}}_i + \dot{\boldsymbol{\theta}}_i^\top \ddot{\boldsymbol{\theta}}_i \\ &+ k_i(\dot{\mathbf{x}}_i - \dot{\boldsymbol{y}}_i)^\top(\mathbf{x}_i - \boldsymbol{y}_i).\end{aligned}$$

Evaluating $\dot{\mathcal{W}}_i$ along the closed-loop system, using **P3** and **A1**, returns

$$\begin{aligned}\dot{\mathcal{W}}_i =& - k_i\dot{\mathbf{q}}_i^\top \mathbf{J}_i^\top(\mathbf{q}_i)\boldsymbol{\Phi}^\top(\boldsymbol{\xi}_i)\left(\mathbf{x}_i - \boldsymbol{y}_i\right) \\ &- k_i\dot{\boldsymbol{\theta}}_i^\top \boldsymbol{\Phi}^\top(\boldsymbol{\xi}_{y,i})(\boldsymbol{y}_i - \mathbf{x}_i) - d_i|\dot{\boldsymbol{\theta}}_i|^2 \\ &- \gamma_i\dot{\boldsymbol{\theta}}_i^\top \boldsymbol{\Phi}^\top(\boldsymbol{\xi}_{y,i})(\boldsymbol{y}_i - \boldsymbol{y}_j(t - T_j(t))) \\ &+ k_i(\dot{\mathbf{x}}_i - \dot{\boldsymbol{y}}_i)^\top(\mathbf{x}_i - \boldsymbol{y}_i),\end{aligned}$$

using the fact that $\dot{\boldsymbol{y}}_{y,i}^\top = \dot{\boldsymbol{\theta}}_i^\top \boldsymbol{\Phi}^\top(\boldsymbol{\xi}_{y,i})$ and relations (3) and (5) with the first term of the last equation and doing some algebra we get

$$\dot{\mathcal{W}}_i = -d_i|\dot{\boldsymbol{\theta}}_i|^2 - \gamma_i\dot{\boldsymbol{\theta}}_i^\top \boldsymbol{\Phi}^\top(\boldsymbol{\xi}_{y,i})(\boldsymbol{y}_i - \boldsymbol{y}_j(t - T_j(t))).$$

Consider now the total energy function

$$\mathcal{W} = \frac{1}{\gamma_l}\mathcal{W}_l + \frac{1}{\gamma_r}\mathcal{W}_r + \frac{1}{2}|\boldsymbol{y}_l - \boldsymbol{y}_r|^2.$$

Evaluating $\dot{\mathcal{W}}$ along ∇_l and ∇_r, returns

$$\begin{aligned}\dot{\mathcal{W}} =& -\frac{d_l}{\gamma_l}|\dot{\boldsymbol{\theta}}_l|^2 - \frac{d_r}{\gamma_r}|\dot{\boldsymbol{\theta}}_r|^2 - \dot{\boldsymbol{y}}_l^\top(\boldsymbol{y}_r - \boldsymbol{y}_r(t - T_r(t))) \\ &- \dot{\boldsymbol{y}}_r^\top(\boldsymbol{y}_l - \boldsymbol{y}_l(t - T_l(t))),\end{aligned}$$

using the facts that $\boldsymbol{y}_i - \boldsymbol{y}_i(t - T_i(t)) = \int_{t-T_i(t)}^t \dot{\boldsymbol{y}}_i(\sigma)d\sigma$ and $|\dot{\boldsymbol{\theta}}_i|^2 = |\dot{\boldsymbol{y}}_i|^2 + 3|\dot{\boldsymbol{\xi}}_{y,i}|^2$ (from **P4** and relation (4)), $\dot{\mathcal{W}}$, can be written as

$$\begin{aligned}\dot{\mathcal{W}} =& -\frac{d_l}{\gamma_l}\left(|\dot{\boldsymbol{y}}_l|^2 + 3|\dot{\boldsymbol{\xi}}_{y,l}|^2\right) - \frac{d_r}{\gamma_r}\left(|\dot{\boldsymbol{y}}_r|^2 + 3|\dot{\boldsymbol{\xi}}_{y,r}|^2\right) \\ &- \dot{\boldsymbol{y}}_l^\top \int_{t-T_r(t)}^t \dot{\boldsymbol{y}}_r(\sigma)d\sigma - \dot{\boldsymbol{y}}_r^\top \int_{t-T_l(t)}^t \dot{\boldsymbol{y}}_l(\sigma)d\sigma.\end{aligned} \tag{10}$$

$\mathcal{W}$ does not qualify as a Lyapunov Function, *i.e.*, it does not satisfy $\dot{\mathcal{W}} < 0$. Then, in the same spirit as in (Nuño et al., 2013), we integrate $\dot{\mathcal{W}}$ from 0 to t and apply Lemma 1 of (Nuño et al., 2009) to the double integral terms with $\lambda_i \in \mathbb{R}_{>0}$. This yields

$$\mathcal{W}(0) \geq \mathcal{W}(t) + \lambda_l\|\dot{\boldsymbol{y}}_l\|_2^2 + \lambda_r\|\dot{\boldsymbol{y}}_r\|_2^2.$$

This last and the fact that $\mathcal{W}(t) \geq 0$, for all $t \geq 0$, ensures that $\mathcal{W} \in \mathcal{L}_\infty$ and $\dot{\boldsymbol{y}}_i \in \mathcal{L}_2$. Besides, using this last, relation (5) and **P6** we can conclude that $\dot{\boldsymbol{\theta}}_i \in \mathcal{L}_2$. Since $\mathcal{W}$ is positive definite and radially unbounded with respect to $\dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_i, |\mathbf{x}_i - \boldsymbol{y}_i|, |\boldsymbol{y}_l - \boldsymbol{y}_r|$ then all these signals are also bounded. Considering that $\dot{\boldsymbol{\theta}}_i \in \mathcal{L}_\infty$ and using relation (5) and **P6** we can conclude that $\dot{\boldsymbol{y}}_i$ is also bounded. Since $\dot{\mathbf{q}}_i$ is bounded using relations (3), (5) and properties **P6** and **P7** we can conclude that $\dot{\mathbf{x}}_i$ is also bounded. Since

$$\mathbf{x}_l - \mathbf{x}_r = \underbrace{\mathbf{x}_l - \boldsymbol{y}_l}_{\mathcal{L}_\infty} + \underbrace{\boldsymbol{y}_l - \boldsymbol{y}_r}_{\mathcal{L}_\infty} - (\underbrace{\mathbf{x}_r - \boldsymbol{y}_r}_{\mathcal{L}_\infty}),$$

then $|\mathbf{x}_l - \mathbf{x}_r| \in \mathcal{L}_\infty$. This establishes Part a) of the proof.

In order to prove Part b), we set $\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$. In this case $\mathcal{E}_i = 0$. Using the same total energy function $\mathcal{W}$ but without the term $(-\mathcal{E}_i + \kappa_i)$, the same boundedness of signals can be established.

The error term $\boldsymbol{y}_i - \boldsymbol{y}_j(t - T_j(t))$ can be written as

$$\boldsymbol{y}_i - \boldsymbol{y}_j(t - T_j(t)) = \boldsymbol{y}_i - \boldsymbol{y}_j + \boldsymbol{y}_j - \boldsymbol{y}_j(t - T_j(t)).$$

Furthermore, similar as in (Nuño et al., 2018), it can be verified that

$$|\boldsymbol{y}_j - \boldsymbol{y}_j(t - T_j(t))|^2 \leq {}^*T_j\|\dot{\boldsymbol{y}}_j\|_2^2.$$

Assumption **A2**, $|\boldsymbol{y}_l - \boldsymbol{y}_r| \in \mathcal{L}_\infty$ and $\dot{\boldsymbol{y}}_i \in \mathcal{L}_2$ ensure that $|\boldsymbol{y}_i - \boldsymbol{y}_j(t - T_j(t))| \in \mathcal{L}_\infty$. Hence, from the closed-loop systems ∇_l and ∇_r, it can be verified that $\ddot{\boldsymbol{\theta}}_i \in \mathcal{L}_\infty$. Barbalat's Lemma allows us to conclude: $\lim_{t\to\infty} \dot{\boldsymbol{\theta}}_i(t) = \mathbf{0}$ and consequently, based on relation (5) and **P6**, $\lim_{t\to\infty} \dot{\boldsymbol{y}}_i(t) = \mathbf{0}$.

Now, differentiating $\ddot{\boldsymbol{\theta}}_i$ yields

$$\begin{aligned}\frac{d}{dt}\ddot{\boldsymbol{\theta}}_i =& - k_i\boldsymbol{\Phi}^\top(\boldsymbol{\xi}_{y,i})(\dot{\boldsymbol{y}}_i - \dot{\mathbf{x}}_j) - k_i\dot{\boldsymbol{\Phi}}^\top(\boldsymbol{\xi}_{y,i})(\boldsymbol{y}_i - \mathbf{x}_j) \\ &- d_i\ddot{\boldsymbol{\theta}}_i - \gamma_i\boldsymbol{\Phi}^\top(\boldsymbol{\xi}_{y,i})\left(\dot{\boldsymbol{y}}_i - (1 - \dot{T}_j)\dot{\boldsymbol{y}}_j(t - T_j(t))\right) \\ &- \gamma_i\dot{\boldsymbol{\Phi}}^\top(\boldsymbol{\xi}_{y,i})(\boldsymbol{y}_i - \boldsymbol{y}_j(t - T_j(t))).\end{aligned} \tag{11}$$

Assumption **A2** ensures that $\dot{T}_j$ is bounded. Since $\dot{\boldsymbol{\Phi}}^\top(\boldsymbol{\xi}_{y,i}) = \mathrm{diag}(\mathbf{0}_3, \frac{1}{2}\dot{\mathbf{U}}^\top(\boldsymbol{\xi}_{y,i}))$, using that $\dot{\boldsymbol{y}}_i \in \mathcal{L}_\infty$, **P5** and **P6** it can be concluded that $\dot{\boldsymbol{\Phi}}^\top(\boldsymbol{\xi}_{y,i}) \in \mathcal{L}_\infty$. Moreover, towards this end, it has been proved that $\dot{\boldsymbol{y}}_i, \dot{\mathbf{x}}_i, \ddot{\boldsymbol{\theta}}_i, |\mathbf{x}_i - \boldsymbol{y}_i|, |\boldsymbol{y}_i - \boldsymbol{y}_j(t - T_j(t))| \in \mathcal{L}_\infty$. Hence $\frac{d}{dt}\ddot{\boldsymbol{\theta}}_i \in \mathcal{L}_\infty$ and therefore, $\ddot{\boldsymbol{\theta}}_i$ is uniformly continuous and, since

$$\lim_{t\to\infty}\int_0^t \ddot{\boldsymbol{\theta}}_i(\sigma)d\sigma = \lim_{t\to\infty}\dot{\boldsymbol{\theta}}_i(t) - \dot{\boldsymbol{\theta}}_i(0) = -\dot{\boldsymbol{\theta}}_i(0),$$

we have that $\lim_{t\to\infty}\ddot{\boldsymbol{\theta}}_i(t) = \mathbf{0}$. Invoking the same arguments, it can be established that $\lim_{t\to\infty}\frac{d}{dt}\ddot{\boldsymbol{\theta}}_i(t) = \mathbf{0}$. Consequently, from (11) and using relation (5), plus the fact that the unit-quaternion norm is always unitary then $\lim_{t\to\infty}\dot{\mathbf{x}}_i(t) = \mathbf{0}$. Using this last fact, **P6** and relation (5), we get $\lim_{t\to\infty}\boldsymbol{\upsilon}_i(t) = \mathbf{0}$ and from relation (3) and property **P7** we conclude that $\lim_{t\to\infty}\dot{\mathbf{q}}_i(t) = \mathbf{0}$.

The fact that $\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_i \in \mathcal{L}_\infty$ and $\lim_{t\to\infty}\dot{\mathbf{q}}_i(t) = \mathbf{0}$ imply, by Barbalat's Lemma, that $\ddot{\mathbf{q}}_i$ is uniformly continuous and hence $\lim_{t\to\infty}|\boldsymbol{\Phi}^\top(\boldsymbol{\xi}_i)(\mathbf{x}_i(t) - \boldsymbol{y}_i(t))| = 0$.

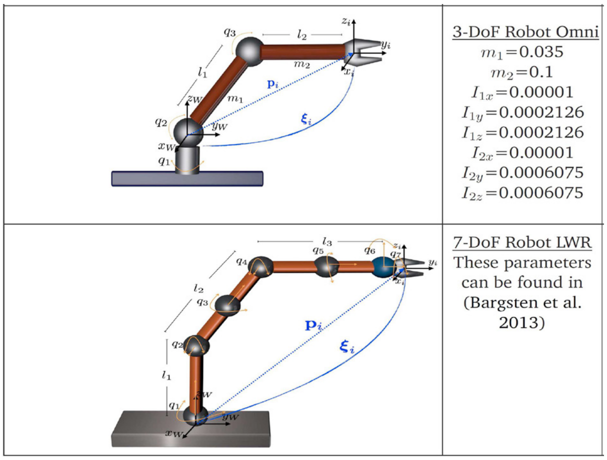


Fig. 2. Robots description and their physical parameters.

Delay	ρ	a_1	ϑ_1(rad/s)	a_2	$\vartheta_2(rad/s)$
T_l	0.04	0.15	7	0.09	27
T_r	0.05	0.10	7	0.16	27

Table 1. Parameters of the time-delays.

Finally, the fact that $\ddot{\boldsymbol{\theta}}_i, \dot{\boldsymbol{\theta}}_i$ and $|\boldsymbol{\Phi}^\top(\boldsymbol{\xi}_i)(\mathbf{x}_i(t) - \boldsymbol{y}_i(t))|$ asymptotically converge to zero, from the closed-loop system and using property **P4**, we can conclude that the only equilibrium solutions are $(\boldsymbol{v}_l, \mathbf{p}_l, \boldsymbol{\xi}_l) = (\mathbf{0}_6, \mathbf{p}_r, \boldsymbol{\xi}_r)$ and $(\boldsymbol{v}_l, \mathbf{p}_l, \boldsymbol{\xi}_l) = (\mathbf{0}_6, \mathbf{p}_r, -\boldsymbol{\xi}_r)$. Using $\mathcal{W}_i$ it can be easily shown that $\boldsymbol{\xi}_l = \boldsymbol{\xi}_r$ corresponds to a minimum energy point and, since $\mathcal{W}(t)$ is a decreasing function, *i.e.*, $\mathcal{W}(0) \geq \mathcal{W}(t)$ for all $t \geq 0$, any perturbation in the other equilibrium point $\boldsymbol{\xi}_l = -\boldsymbol{\xi}_r$ will drive the system to $\boldsymbol{\xi}_l = \boldsymbol{\xi}_r$. Hence, $(\boldsymbol{v}_l, \mathbf{p}_l, \boldsymbol{\xi}_l) = (\mathbf{0}_6, \mathbf{p}_r, \boldsymbol{\xi}_r)$ is asymptotically stable everywhere except at the unstable equilibrium point $(\boldsymbol{v}_l, \mathbf{p}_l, \boldsymbol{\xi}_l) = (\mathbf{0}_6, \mathbf{p}_r, -\boldsymbol{\xi}_r)$. This concludes the proof.

4. SIMULATIONS

The simulations are carried on using Matlab® and Simulink® version 9.3. Fig. 1 shows the teleoperation system composed of heterogeneous robots in which the control algorithm detailed in Section 3 is simulated. At the local site there is an OMNI robot (3-DoF) and at the remote site a LWR robot (7-DoF). In Fig. 2 it can be observed a description of the robots used in the network and the values of their physical parameters. The direct kinematics, the Jacobians, and the dynamic models of the robots are detailed in (Aldana, 2015; Bargsten et al., 2013). The algorithm defined in the paper (Spurrier, 1978) is used to derive the unit-quaternions from the rotation matrices.

The variable time-delays between the local and remote site are given by $T_{ij} = \rho + a_1 sin(\vartheta_1 t) + a_2 sin(\vartheta_2 t)$. In Table 1 are listed the values of the parameters used in the simulations for the interconnection's delays. The Fig.3 shows a small time window of them.

The controller parameters used in this simulation, that fulfill condition (9), can be seen in Table 2. The performance of the proposed control scheme is tested under the following situation: The local and the remote robots have different initial poses and no external forces are injected ($\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$). It can be observed in Fig. 4.(a) and Fig.4.(b) that it takes 10 seconds to the robots to converge to a common pose ($\mathbf{x_l} = \mathbf{x_r}$). Fig. 5.(a) and Fig. 5.(b) show the linear and angular velocities for each robot, and it is observed that they converge to zero as expected.

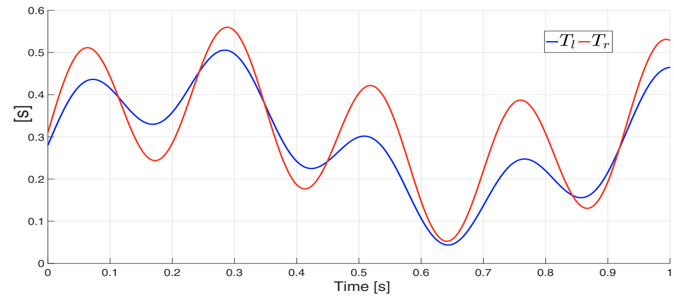
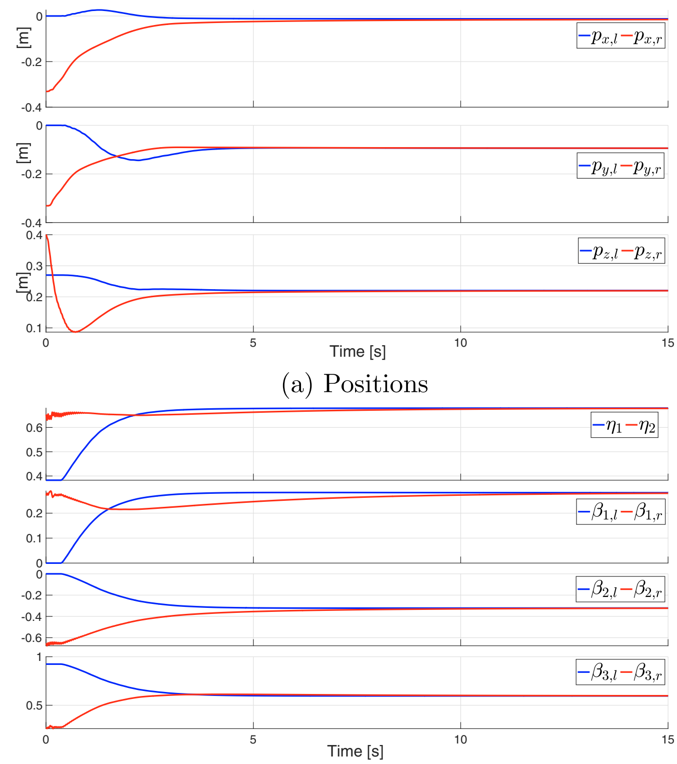


Fig. 3. Sample of the variable time-delays.

	k_i	d_i	γ_i
Local Robot (l)	325	25	55
Remote Robot (r)	325	25	38

Table 2. Controller Parameters



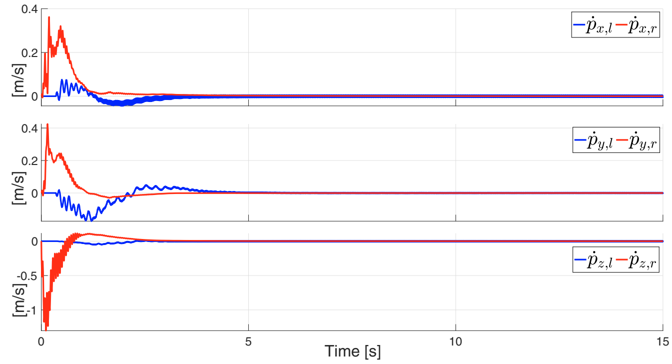
(a) Positions

(b) Unit-quaternions (Orientations)

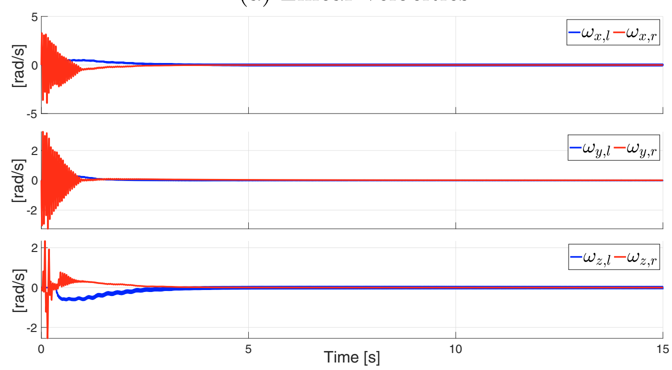
Fig. 4. Pose of the robots that compose the teleoperation system.

5. CONCLUSION

In this paper, teleoperation systems composed of heterogeneous robots, without velocity sensors and with variable time-delays in the interconnection are analyzed. For these systems, a control scheme is proposed and it was proved that under the assumptions that the human operator and the environment define passive maps from force to velocity, the velocities and pose errors between the local and the remote manipulators are bounded. Moreover, in the case that the human and the environment forces are zero, the velocities and pose errors converge asymptotically



(a) Linear velocities



(b) Angular velocities

Fig. 5. Velocities of the robots that compose the teleoperation system.

to zero. The proposed approach employs, the singularity-free, unit-quaternions to represent the orientation of the end-effectors. Simulations results are shown to support the theoretical results of this paper. Future work will consider the extension of this controller to solve the pose consensus problem in heterogeneous robot networks and the experimental validation of these theoretical results.

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